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USING A HOT-WIRE ANEMOMETER FOR MEASUREMENT OF CHARACTERISTICS OF A RANDOM ACOUSTIC FIELD IN COMPRESSIBLE FLOWS

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The problem of interpretation of hot-wire measurements of acoustic fields in compressible flows is considered. Relations between mass-flow and total-temperature fluctuations registered by a hot-wire anemometer and pressure and velocity fluctuations are found. The relations obtained are applicable in the general case for measurement of resultant acoustic fluctuations at some point of the flow, which are generated by arbitrary distributed sources of sound with prior-unknown properties.

Pressure fluctuations are usually measured by industrial microphones or special microphones made for solving particular problems. Various types of microphones have been developed: capacitor, electrodynamic, piezoelectric, magnetostrictive, optical, etc. Each type of microphones has disadvantages, especially if the measurements are performed in flows and the ambient pressure differs from atmospheric. A hot-wire anemometer can also be used to measure pressure fluctuations in gas flows. This method is especially efficient in high-speed flows, because the probes are small, and consequently, their disturbing influence on the flow investigated is insignificant. Moreover, in addition to information about the intensity of pressure fluctuations, the output signal of the hot-wire anemometer often includes information on the direction of propagation of acoustic waves and the position and velocity of the sources of pressure fluctuations [1–3].

It was first shown in [1] that, using the method of fluctuation diagrams obtained on the basis of measurement at several (minimum three) overheatings of the hot-wire probes, one can divide the total field of fluctuations in a supersonic flow into vorticity, entropy, and acoustic modes, and obtain correlations between disturbances of different types. Other possibilities of the hot-wire anemometry used to measure acoustic fluctuations at supersonic flow velocities were demonstrated in [2, 3]. In [4], another approach to interpretation of hot-wire measurement data was proposed, but the main relations of this method could be easily obtained from the relations [1] by simple substitution of variables.

The hot-wire method for measurement of acoustic fluctuations in compressible subsonic flows was further developed in [5–7]. In addition to general relations for arbitrarily distributed sources of acoustic fluctuations, analytical relations were obtained for interpretation of results of hot-wire measurements in the near acoustic field (from a localizated source of spherical waves) and in the far field (plane waves), and also for uniformly distributed sources of identical intensity. It was shown that the hot wire can be used to determine the intensity of pressure fluctuations and associated velocity fluctuations, correlation coefficients, and other characteristics. The correlations obtained were verified by numerous wind-tunnel measurements [8]. At the same time, in the case of arbitrarily distributed sources of different intensity, the hot-wire anemometer can only be used to obtain direct information on the intensity of mass-flow and total-temperature fluctuations and the correlation coefficient between them.

The problem of determining the acoustic field characteristics, first of all, pressure fluctuations, using hot-wire probes in the case of arbitrarily distributed sources of different intensity is considered in the present paper. The problems of mode separation and calibration of hot-wire probes for determining sensitivity coefficients are not analyzed here, since they are described in much detail in [9]. The type of the anemometer used (constant current, constant temperature, or constant voltage) is not taken into account, because it has no principle influence on the measurement results.

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1. Basic Initial Relations. The relation between flow fluctuations (mass flow rate $m = \rho u$ and total temperature T_0) and the anemometer output (voltage on the probe e) is determined by the equation

$$e'/e = \pm(Fm'/m - GT_0'/T_0), \tag{1.1}$$

where the fluctuating values are primed, and F and G are the sensitivity coefficients to the mass flow and total temperature, respectively. The sign before the bracket depends on the type of the anemometer used but, as is shown below, is not important for further analysis. Squaring the left and right sides of Eq. (1.1), averaging, and passing to the variables $\vartheta = \langle e\rangle/G$ and $r = F/G$, we obtain the equation of the fluctuation diagram [1]

$$\vartheta^2(r) = r^2\langle m\rangle^2 - 2rR_{mT_0}\langle m\rangle\langle T_0\rangle + \langle T_0\rangle^2, \tag{1.2}$$

where the root-mean-square fluctuations nondimensionalized by the mean values of the corresponding flow parameters are given in broken brackets and $R_{mT_0} = \langle mT_0\rangle/(\langle m\rangle\langle T_0\rangle)$ is the correlation coefficient between the mass-flow and total-temperature fluctuations; the relative sensitivity coefficient r is assumed to be known from calibrations.

The fluctuation diagram $\vartheta(r)$ is a part of a hyperbola located in the first quadrant. The horizontal axis of symmetry of this hyperbola is the r axis.

It is convenient to use the relation between voltage fluctuations at the hot-wire probe and fluctuations of velocity u, density ρ, and temperature T for analyzing fluctuation modes. For the mass-flow fluctuations, this relation has the form

$$m'/m = \rho'/\rho + u'/u. \tag{1.3}$$

The expression for the total temperature fluctuations follows from the relation

$$T_0 = T[1 + (\gamma - 1)\mathrm{M}^2/2] = T + u^2/(2c_p),$$

which yields the following equation after simple transformations:

$$T_0'/T_0 = \alpha T'/T + \beta u'/u. \tag{1.4}$$

Here γ is the ratio of specific heats, c_p is the specific heat of the gas at constant pressure, and α and β are known functions of the Mach number M:

$$\alpha = \frac{T}{T_0} = \Big(1 + \frac{\gamma-1}{2}\mathrm{M}^2\Big)^{-1}, \quad \beta = (\gamma-1)\mathrm{M}^2\Big(1 + \frac{\gamma-1}{2}\mathrm{M}^2\Big)^{-1} = 2(1-\alpha). \tag{1.5}$$

Substituting (1.3) and (1.4) into (1.1), one can write the equation relating fluctuations of the hot-wire voltage to fluctuations of flow parameters, in general case, in the form

$$\frac{e'}{eG} = \pm\Big(r\,\frac{m'}{m} - \frac{T_0'}{T_0}\Big) = \pm\Big[r\Big(\frac{\rho'}{\rho} + \frac{u'}{u}\Big) - \alpha\,\frac{T'}{T} - \beta\,\frac{u'}{u}\Big]. \tag{1.6}$$

Equation (1.6) can be transformed for the particular case of the acoustic mode of disturbances as follows. The relations

$$\frac{\rho'}{\rho} = \frac{1}{\gamma}\frac{p'}{p}, \qquad \frac{p'}{p} = \frac{\rho'}{\rho} + \frac{T'}{T}, \qquad \frac{T'}{T} = \frac{\gamma-1}{\gamma}\frac{p'}{p}$$

follow from the equation of adiabat $p/\rho^\gamma = \mathrm{const}$ and the equation of state $p = \rho RT$, where R is the universal gas constant.

Substituting these expressions into (1.6) for the particular case of the acoustic mode, we obtain the equation

$$\frac{e'}{eG} = \pm\Big[r\Big(\frac{1}{\gamma}\frac{p'}{p} + \frac{u'}{u}\Big) - \beta\Big(\frac{1}{\gamma\mathrm{M}^2}\,\frac{p'}{p} + \frac{u'}{u}\Big)\Big], \tag{1.7}$$

which related the instantaneous values of voltage fluctuations at the hot-wire probe to the instantaneous fluctuations in the acoustic wave. It should be noted that u' in (1.7) are not velocity fluctuations in the acoustic wave but projections of velocity fluctuations in the acoustic wave onto the mean-velocity vector. Therefore, in contrast to the known works [1, 2], we do not express u' via the velocity fluctuations in the acoustic wave v', because such a change leads to the appearance of an additional quantity, the angle χ between the direction of propagation of the acoustic wave and the mean-velocity vector ($u' = v'\cos\chi$). As is shown in Sec. 2, this allows one to solve the posed problem of determining the intensity of pressure fluctuations in a random acoustic field, whereas the solutions in [1, 2] were obtained only for supersonic velocities under the assumption $\chi = \mathrm{const}$.

2. Relations for the Root-Mean-Square Fluctuations. The fluctuation diagram described by Eq. (1.2) can be constructed according to the results of fluctuation measurements. Using this diagram, one can calculate the root-mean-square fluctuations of the mass flow $\langle m\rangle$ and total temperature $\langle T_0\rangle$ and the correlation coefficient between them R_{mT_0}.

We square and average the left and right sides of Eq. (1.7) in order to determine the intensity of pressure fluctuations in the acoustic wave; as a result, we obtain

$$\vartheta^2(r) = r^2\Big(\frac{\langle p\rangle^2}{\gamma^2} + \frac{2}{\gamma}\,R_{up}\langle u\rangle\langle p\rangle + \langle u\rangle^2\Big) - 2r\beta\Big(\langle u\rangle^2 + \frac{1}{\gamma}\Big(1 + \frac{1}{\mathrm{M}^2}\Big)R_{up}\langle u\rangle\langle p\rangle + \frac{1}{\gamma^2\mathrm{M}^2}\,\langle p\rangle^2\Big)$$
$$+ \beta^2\Big(\langle u\rangle^2 + \frac{2}{\gamma\mathrm{M}^2}\,R_{up}\langle u\rangle\langle p\rangle + \frac{1}{\gamma^2\mathrm{M}^4}\,\langle p\rangle^2\Big). \tag{2.1}$$

Equating the terms at the same powers of r in Eqs. (1.2) and (2.1), we obtain the following system of equations relating the mass-flow and total-temperature fluctuations and the correlation coefficient between them to the pressure and velocity fluctuations and the correlation coefficient between them:

$$\begin{aligned}\langle m\rangle^2 &= \frac{\langle p\rangle^2}{\gamma^2} + \frac{2}{\gamma}\,R_{up}\langle u\rangle\langle p\rangle + \langle u\rangle^2,\\ \frac{R_{mT_0}\langle m\rangle\langle T_0\rangle}{\beta} &= \langle u\rangle^2 + \frac{1}{\gamma}\Big(1 + \frac{1}{\mathrm{M}^2}\Big)R_{up}\langle u\rangle\langle p\rangle + \frac{\langle p\rangle^2}{\gamma^2\mathrm{M}^2},\\ \frac{\langle T_0\rangle^2}{\beta^2} &= \langle u\rangle^2 + \frac{2}{\gamma\mathrm{M}^2}\,R_{up}\langle u\rangle\langle p\rangle + \frac{\langle p\rangle^2}{\gamma^2\mathrm{M}^4}.\end{aligned} \tag{2.2}$$

Solving system (2.2) with respect to $\langle p\rangle$, $\langle u\rangle$, and R_{up}, we obtain the relations

$$\langle p\rangle^2 = \frac{\gamma^2\mathrm{M}^4\langle m\rangle^2}{(\mathrm{M}^2-1)^2} - \frac{2\gamma^2\mathrm{M}^4R_{mT_0}\langle m\rangle\langle T_0\rangle}{\beta(\mathrm{M}^2-1)^2} + \frac{\gamma^2\mathrm{M}^4\langle T_0\rangle^2}{\beta^2(\mathrm{M}^2-1)^2},$$
$$\langle u\rangle^2 = \frac{\langle m\rangle^2}{(\mathrm{M}^2-1)^2} - \frac{2\mathrm{M}^2R_{mT_0}\langle m\rangle\langle T_0\rangle}{\beta(\mathrm{M}^2-1)^2} + \frac{\mathrm{M}^4\langle T_0\rangle^2}{\beta^2(\mathrm{M}^2-1)^2},$$
$$R_{up} = \frac{1}{\vartheta(\beta)}\,\frac{R_{mT_0}\beta\langle m\rangle\langle T_0\rangle(\mathrm{M}^2+1) - \mathrm{M}^2\langle T_0\rangle^2 - \beta^2\langle m\rangle^2}{\sqrt{\beta^2\langle m\rangle^2 + 2\mathrm{M}^2R_{mT_0}\beta\langle m\rangle\langle T_0\rangle + \mathrm{M}^4\langle T_0\rangle^2}}$$

or, in a more compact form,

$$\langle p\rangle = \frac{\vartheta(\beta)}{\beta}\,\frac{\gamma\mathrm{M}^2}{\mathrm{M}^2-1}; \tag{2.3}$$

$$\langle u\rangle = \sqrt{\frac{\langle m\rangle^2}{1-\mathrm{M}^2} + \frac{\vartheta^2(\beta)\mathrm{M}^2}{\beta^2(\mathrm{M}^2-1)^2} + \frac{\langle T_0\rangle^2\mathrm{M}^2}{\beta^2(\mathrm{M}^2-1)}}; \tag{2.4}$$

$$R_{up}\langle u\rangle\langle p\rangle = \frac{\gamma\mathrm{M}^2}{\mathrm{M}^2-1}\Big(\frac{R_{mT_0}\langle m\rangle\langle T_0\rangle}{\beta} - \frac{\langle T_0\rangle^2}{\beta^2} - \frac{\vartheta^2(\beta)}{\beta^2(\mathrm{M}^2-1)}\Big). \tag{2.5}$$

These equations allow one to determine the intensity of pressure fluctuations and other characteristics of the acoustic field using the measured values of $\langle m\rangle$, $\langle T_0\rangle$, and R_{mT_0}. The values of $\vartheta(\beta)$ in Eqs. (2.3)–(2.5) are calculated by (1.2) for $r = \beta$ [see (1.5)]. This quantity is determining for the intensity of pressure fluctuations $\langle p\rangle$ [see (2.3)]. As is known, the hot-wire anemometer probe is insensitive to the direction of propagation of acoustic disturbances for overheatings corresponding to $r = \beta$ [5].

As $\mathrm{M} \to 0$, system (2.3)–(2.5) yields the simple relations

$$\langle p\rangle = \frac{\gamma}{\gamma-1}\,\langle T_0\rangle, \quad \langle u\rangle = \sqrt{\langle m\rangle^2 - \frac{2R_{mT_0}\langle m\rangle\langle T_0\rangle}{\gamma-1} + \frac{\langle T_0\rangle^2}{(\gamma-1)^2}} = \frac{\vartheta(\gamma-1)}{\gamma-1},$$

$$R_{up}\langle u\rangle\langle p\rangle = \gamma\Big(\frac{R_{mT_0}\langle m\rangle\langle T_0\rangle}{\gamma-1} + \frac{\langle T_0\rangle^2}{(\gamma-1)^2}\Big).$$

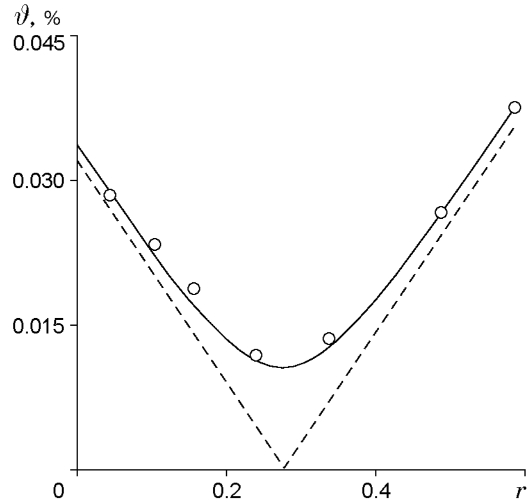

Fig. 1.

3. Verification of the Relations Obtained. Numerous measurements in test sections of wind tunnels at high subsonic velocities showed that the corresponding fluctuation diagrams have the form of hyperbolas. Since the presence of the vorticity or entropy mode in the flow is little probable, as a rule, because of the great flow compression behind the plenum chamber, acoustic disturbances generated by different sources make the main contribution to the overall diagram. The most significant among them are the perforation holes of the test-section walls, the boundary layer on the test-section walls, steps, jets at the ventilated test-section, noise of ejectors and fans, etc.

Intensity and spectra of fluctuations generated by each source of disturbances depend on various factors. Since the distance between the sources and measurement points and their positions are different, the overall intensity and fluctuation spectra in the test section in this case are a superposition of a great number of fluctuation processes. Figure 1 shows the diagram of fluctuations measured by a hot-wire probe at the axis of the wind-tunnel test section at a free-stream Mach number M $= 0.71$. The values of mass-flow fluctuations $\langle m \rangle = 0.11\%$, total-temperature fluctuations $\langle T_0 \rangle = 0.034\%$, and the correlation coefficient between them $R_{mT_0} = 0.96$ are calculated by the least-squares technique using the parameters of the hyperbola (solid curve in Fig. 1) drawn through the experimental points. The asymptotic lines of the hyperbola are shown by the dashed curves. Besides, the pressure fluctuations at the wall $\langle p \rangle = 0.13\%$ were measured using the Kistler pressure probe located on the test-section wall in the same cross section where the hot-wire anemometer was placed. The acoustic field characteristics calculated according to the hot-wire measurements using the above relations are $\langle p \rangle = 0.12\%$ and $\langle u \rangle = 0.07\%$. The calculated value of the correlation coefficient R_{up} is close to zero. Thus, the intensities of pressure fluctuations calculated using the fluctuation diagrams and the above relations are in good agreement with the Kistler probe measurements.

The results obtained allow one to interpret correctly hot-wire measurements, when acoustic fluctuations are generated by arbitrarily distributed sources. In many cases, these results may be efficient when other fluctuations modes are present in the flow. Thus, as is seen above, first, the hot-wire probe is insensitive to the direction of propagation of acoustic disturbances, second, the acoustic mode intensity is completely determined by the fluctuation-diagram values at overheatings corresponding to $r = \beta$ [see Eq. (2.3)]. However, just at this value of r is the hot-wire anemometer insensitive to the vorticity mode, and its presence is not important in determining the acoustic mode intensity. When the entropy mode is present in the flow, its intensity and possible influence on the measurement results should be evaluated. In many cases, the entropy mode is rather low as compared to the acoustic one. Besides, the spectral composition of the modes is usually different. This circumstance can be used for separation of modes, if some mode is assumed to prevail in considering different spectrum parts.

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